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1) (a) We know that $Y|X \sim \text{Binomial}(n=20, p=\frac{X}{10})$

10 Hence, $E(Y|X) = n \cdot p = \frac{20 \cdot X}{10} = \boxed{2X}$

(b) $E(Y) = E(E(Y|X))$ By the double-expectation formula.

$= E(2X)$ By def. of $E(Y|X)$

$= 2E(X)$ By linearity

$= 2\left(\frac{10+0}{2}\right)$ Since $X \sim \text{Uniform}(\{0,1,\dots,10\})$.

$= \boxed{10}$

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2) (a) $E(2X-3Y) = 2E(X) - 3E(Y)$ By Linearity

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$= 2 \cdot 1 - 3 \cdot 2$ By Hypothesis.

$= \boxed{-4}$

(b) $Var(2X-3Y) = Var(2X) + Var(-3Y)$ Since X, Y are independent

$= 2^2 Var(X) + (-3)^2 Var(Y)$ By property of Var.

$= 4 \cdot 1 + 9 \cdot 4$ By Hypothesis

$= \boxed{40}$

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4) Let X = # of results which do not appear on any of eight dice.

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Define: $X_i = \begin{cases} 1 & \text{if number } i \text{ does not appear on any of eight dice.} \\ 0 & \text{otherwise} \end{cases}$

then: $X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$.

Since X_i is an indicator, $E(X_i) = P(\text{number } i \text{ not appearing on any of eight dice})$

$= \left(\frac{5}{6}\right)^8$ (Since each die is independent), $1 \le i \le 6$

therefore, $E(X) = E\left(\sum_{i=1}^{6} X_i\right)$

$= \sum_{i=1}^{6} E(X_i)$ By Linearity

$= 6 \cdot \left(\frac{5}{6}\right)^8 = \boxed{\frac{5^8}{6^7}}$

9

5) Let X = # of hours that a light bulb works before burning out;
Note $X \geq 0$; for any x. We want to estimate $P(X \geq 1{,}200)$. If:

(a) $E(X) = 1{,}000$. Without any further assumptions we can use the Markov's Inequality: $P(X > b) \leq \frac{E(X)}{b}$

$$P(X > 1{,}199) = P(X \geq 1{,}200) \leq \boxed{\frac{1{,}000}{1{,}199}}$$

$P(x > 1200) = P(x \geq 1200)$

x is continuous.

1200

-0.5

(b) $E(X) = 1{,}000$ and $Var(X) = 10^4$.
Here we can use Chebychev's Inequality. $P(|X^*| \geq d) \leq \frac{1}{d^2}$

$$P(X \geq 1{,}200) = P\left(\frac{X-\mu}{\sigma} \geq \frac{1{,}200 - 1{,}000}{\cancel{10^4}\ 2}\right)$$

-1

$$= P\left(X^* \geq \frac{200}{10^2\ 10^2}\right)$$

$$= P(X^* \geq 0.02) \leq \frac{1}{(0.02)^2} > 1 \Rightarrow \text{the estimate is useless.}$$

(c) $E(X) = 1{,}000$ and $Var(X) = 10^4$.
Here we can tight chebychev's Inequality by a factor of $\frac{1}{2}$.
Hence,

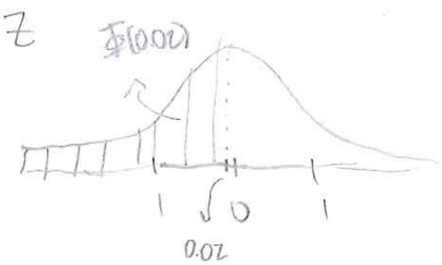
previously calculated

$$P(X \geq 1{,}200) = P(X^* \geq 0.02) \leq \frac{1}{2 \times (0.02)^2} > 1 \Rightarrow \text{Still, a useless estimate.}$$

(d) If X is approximately Normal with $E(X) = 1{,}000$ and $Va(X) = 10^4$, then

$$P(X > 1{,}200) = 1 - P(X \leq 1{,}200) = 1 - P(X^* \leq 0.02)$$

$$= 1 - \Phi(0.02)$$



this is actually close to 0.5 (50%)
since 0.02 is close to 0.
So an estimate would be that 0.5.

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(2)

Let $X_i \sim \text{Poisson}(\mu)$. We want to find μ such that: 9

If no more than 0.1 % of cookies are to have no chips at all, this is equivalent of saying that on average one cookie will have no chips with probability 0.01. Hence,

$$P(X_i = 0) = 0.001 \overset{X_i \sim Poi(\mu)}{\iff} \frac{e^{-\mu} \cdot \mu^0}{0!} = 0.01 \iff e^{-\mu} = 0.001$$

-1

$$\Rightarrow \ln(e^{-\mu}) = \ln(0.01)$$

$$\Rightarrow -\mu = \ln(0.01) \Rightarrow \boxed{\mu = -\ln(0.01)}$$

Note that this is a positive number since $0 < 0.01 < 1$, so $\ln(0.01) < 0$.